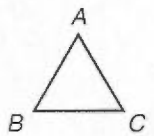
Lesson 25 GED SKILL Triangles and Quadrilaterals

Triangles



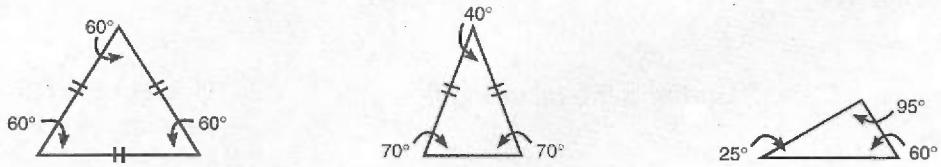
A triangle has three sides and three angles and is named by its vertices (in any order). Identify a side of a triangle by the two letters that name the vertices at the ends of the side. In $\triangle ABC$, the sides are $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$.

A triangle is named by the lengths of its sides and the measures of its angles. The triangles below are named by the lengths of their sides.

Equilateral triangle
All sides and angles are congruent; each angle measures 60°.

Isosceles triangle
Two sides and the two angles opposite these sides are congruent.

Scalene triangle
No sides and no angles are congruent.



TIP

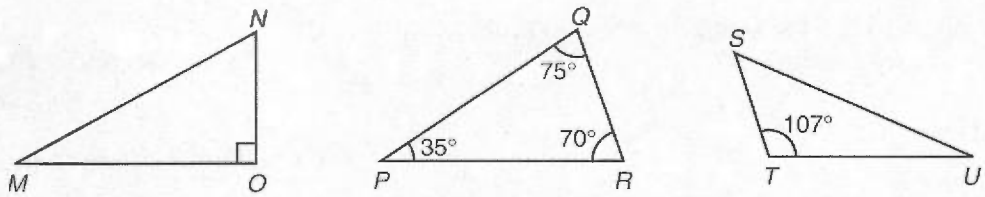
The symbols I, II, and III marked on a side show which sides are congruent. Remember: *Congruent* means *equal*.

The following triangles are named by the measures of their angles.

Right triangle
One angle is a right angle (equal to 90°).

Acute triangle
All three angles are acute (less than 90°).

Obtuse triangle
One angle is obtuse (greater than 90°).



TIP

Questions on the GED Mathematics Test may include more information than you need to solve a problem. Use only the information you need based on what the question asks.

In any triangle, the sum of the measures of the angles is 180°. (Note: A triangle can have only one right or obtuse angle. The other two angles must be acute.)

Example In a right triangle, one acute angle is twice the measure of the other acute angle. What is the measure of the larger angle?

Step 1 **Make a sketch.** A right triangle has one right angle ($m = 90°$).

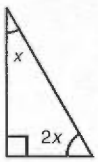
Step 2 **Assign the unknowns.**

Let x = the measure of the smaller acute angle.
Let $2x$ = the measure of the larger acute angle.

Step 3 **Write an equation.** $x + 2x + 90° = 180°$

Step 4 **Solve.** $3x + 90° = 180°$

$3x = 90°$; so $x = 30°$ and $2x = 60°$

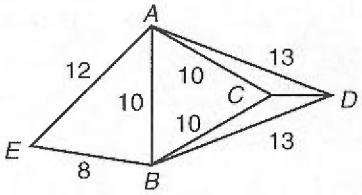


The larger acute angle measures **60°.**

GED SKILL FOCUS

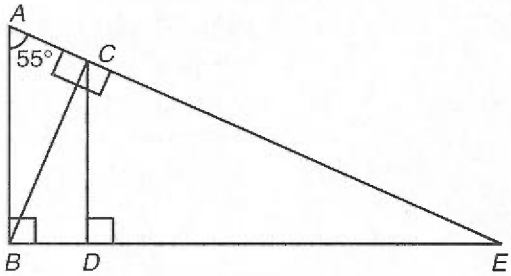
A. Solve.

Refer to the figure at right to answer Questions 1 through 5. (*Hint*: The figure contains 5 triangles.)



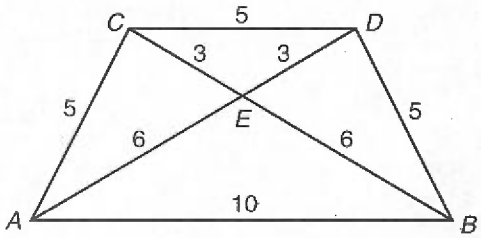
1. Name an equilateral triangle.
2. Name an isosceles triangle.
3. Name an obtuse triangle.
4. Name an acute triangle.
5. Name a scalene triangle.

Refer to the figure at right to answer Questions 6 through 14.



6. Name the triangle that has a right angle at B.
7. Name two triangles that have right angles at D.
8. Name two triangles that have right angles at C.
9. If $m\angle A = 55°$, then what is $m\angle E$?
10. What is $m\angle ABC$?
11. What is $m\angle DCE$?
12. What is $m\angle CBD$?
13. What is $m\angle BCD$?
14. How are the five triangles in the figure alike?

Refer to the figure at right to answer Question 15.



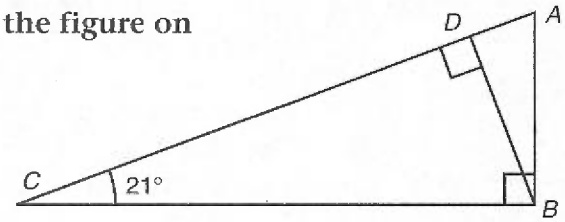
15. How many isosceles triangles are there in the figure? (*Hint:* The figure contains 8 triangles.)

_____

16. In a right triangle the measure of one acute angle is 4 times the measure of the other acute angle. What is the measure of the smaller angle?

17. In a triangle the measure of one angle is 16° more than the measure of the smallest angle, and the measure of the third angle is twice the measure of the smallest angle. What are the three angle measures?

B. Write *True* or *False* for each statement. Refer to the figure on the right to answer Questions 18 through 23.



____ 18. $\triangle ABC$ is an equilateral triangle.

____ 19. $\triangle ABC$ is an obtuse triangle.

____ 20. $m\angle BAC$ is equal to $m\angle CBD$.

____ 21. $m\angle ABD$ is less than $m\angle ACB$.

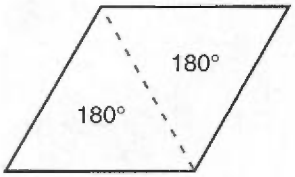
____ 22. $\triangle ABC$ is an acute triangle.

____ 23. There are exactly three right triangles in the figure.

Answers start on page 440.

Quadrilaterals

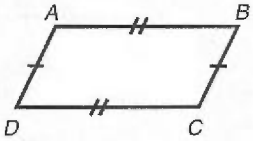
A **quadrilateral** is any geometric figure with four (*quad*) sides (*lateral*). A figure with four sides also has four angles, and the sum of these four angle measures always equals 360°.



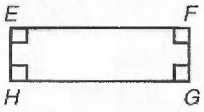
To prove this is so, draw any quadrilateral and draw a line segment called a **diagonal** between the vertices of two opposite angles. The diagonal divides the quadrilateral into two triangles. You know that the sum of the three angles in a triangle is 180°. Since there are two triangles, the sum of the angles in the quadrilateral must be 180° + 180° = 360°.

The following quadrilaterals appear on the GED Mathematics Test.

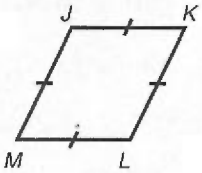
Parallelogram
opposite sides are parallel and congruent; opposite angles are of equal measure·



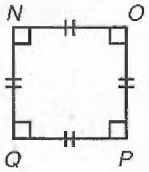
Rectangle
special parallelogram with four right angles



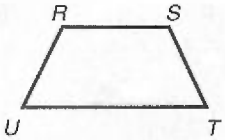
Rhombus
special parallelogram with four sides of equal length



Square
special parallelogram/rhombus/rectangle with four sides of equal length and four right angles



Trapezoid
only one pair of parallel sides (called bases)

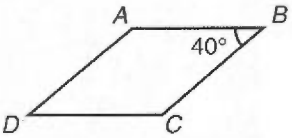


TIP

Write algebraic expressions on the drawing to remember what the variables represent.

On the GED Mathematics Test, you must combine your knowledge of algebra and geometric principles to solve problems.

Example Angle B of the rhombus shown at the left measures 40°. What is the measure of $\angle C$?



Step 1	**Identify known values.**	$m\angle B = 40°$
Step 2	**Identify known relationships.**	sum of angles = 360°
	Opposite angles are equal.	$m\angle D = 40°$; $m\angle C = m\angle A$
Step 3	**Assign the unknown values.**	Let $x = m\angle C$ (and $m\angle A$)
Step 4	**Write an equation.**	$m\angle A + m\angle B + m\angle C + m\angle D = 360°$
Step 5	**Substitute known values.**	$x + 40° + x + 40° = 360°$
		$2(40°) + 2x = 360°$
Step 6	**Solve for unknown values.**	$80° + 2x = 360°$
		$2x = 280°$; $x = 140°$

The measure of $\angle C$ is **140°.**

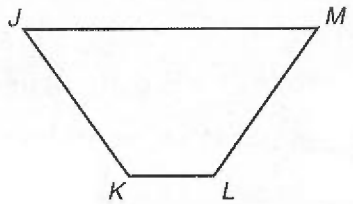
GED SKILL FOCUS

A. Give all possibilities for the names of the following four-sided figures.

1. All sides have lengths of 10 inches.
2. All corners are right angles.
3. Opposite sides are parallel.
4. Only two sides are parallel.
5. There are no right angles.
6. Only one pair of opposite sides is equal in length.

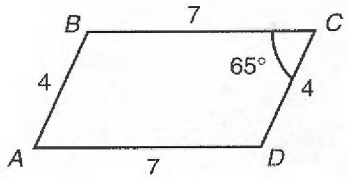
B. Solve.

Refer to the figure below to answer Questions 7 through 9.



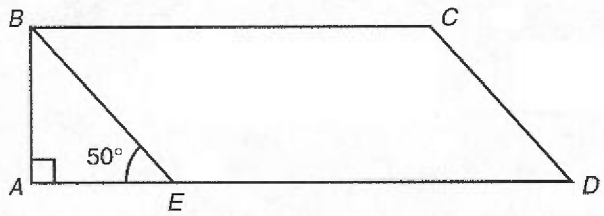
7. Side $JM \parallel$ side KL. What is the most common name of this figure?
8. What is the sum of the four inside angle measures?
9. $m\angle K = m\angle L$ and $m\angle J = m\angle M$. If the measure of $\angle K$ is 125°, what is the measure of $\angle J$?

Refer to the figure below to answer Questions 10 through 12.



10. In the figure, $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. What is the most common name of this figure?
11. What is the measure of $\angle A$?
12. What is the measure of $\angle D$?
13. Quadrilateral $ABCD$ has two pairs of parallel opposite sides. Angle A measures 90°. What are the possible names of the quadrilateral?
14. In a quadrilateral, one angle is 20° less than another angle. The remaining two angles are right angles. What are the measures of the four angles?
15. The lengths of the sides of a four-sided figure, in order, are 8, 12, 8, and 12. There are no right angles, and opposite sides are parallel. What is the figure?

Refer to the figure to answer Questions 16 through 18.



16. Quadrilateral $BCDE$ is a parallelogram. What two things must be true about sides BC and ED?
17. What is the measure of $\angle C$?
18. Quadrilateral $ABCD$ is a trapezoid. What is the measure of $\angle ABC$?

Answers start on page 441.

Congruent Figures

TIP

Always follow steps of logical reasoning to solve for the needed information.

Congruent figures are the same shape and size. One way to find out whether two figures are congruent is to place one figure on top of the other to see whether the sides and angles align perfectly. You may even be able to tell whether figures are congruent simply by looking at them. However, on the GED Mathematics Test, you need to be able to do more than identify which figures "look" congruent; you must be able to prove that the two figures are congruent.

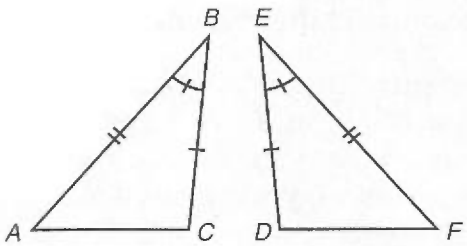
Let's look at triangles. Congruent triangles are exactly the same shape and size. They have matching, or corresponding, vertices; sides; and angles. Marks are used to show which parts correspond.

There are three rules used to prove that two triangles are congruent. Two triangles are congruent if any one of the following rules is true:

RULE 1 Three sides (SSS) are congruent.

RULE 2 Two sides and the angle between them (SAS) are congruent.

RULE 3 Two angles and the side between them (ASA) are congruent.



Example 1 Are triangles *ABC* and *FED* congruent?

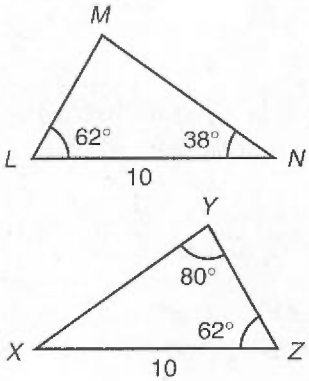
Step 1 **Identify the given corresponding congruent parts.** $\overline{AB} \cong \overline{FE}$, $\overline{BC} \cong \overline{ED}$; $\angle B \cong \angle E$

Step 2 **Identify what relationships are known.** There are two pairs of corresponding congruent sides. The angles between the congruent sides are also congruent. (SAS)

Step 3 **Identify additional information needed.** None.

Step 4 **Draw a conclusion.** Rule 2 is true; the triangles are congruent.

The triangles are congruent. $\boldsymbol{\Delta ABC \cong \Delta FED}$



Example 2 Is $\triangle LMN$ congruent to $\triangle ZYX$?

Step 1 **Identify the given congruent parts.** $\angle L \cong \angle Z$; $\overline{LN} \cong \overline{ZX}$

Step 2 **Identify known relationships.** $\triangle LMN$: the measures of two angles and the length of the side between them (ASA) $\triangle ZYX$: the measure of one angle and the length of the adjacent side corresponding with $\triangle LMN$

Step 3 **Identify needed information.** measure of $\angle X$

Step 4 **Solve for needed information.** Sum of the measures of the angles of a triangle = 180°, so $m\angle X + 80° + 62° = 180°$

$m\angle X = 180° - 80° - 62° = 38°$

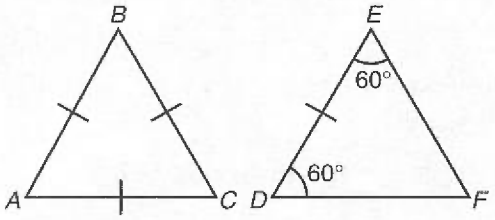
Step 4 **Draw a conclusion.** $m\angle X = 38°$; so $\angle X$ is congruent to $\angle N$ Rule 3 (ASA) is true; the triangles are congruent.

The triangles are congruent. $\boldsymbol{\Delta LMN \cong \Delta ZYX}$

GED SKILL FOCUS

A. Decide whether each pair of triangles is congruent or not. Explain your reasoning.

1.



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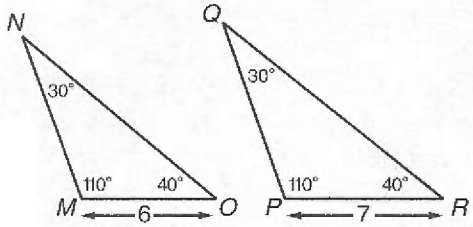
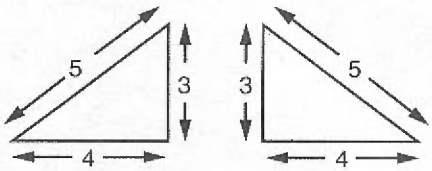


3.



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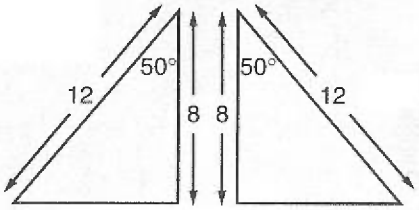
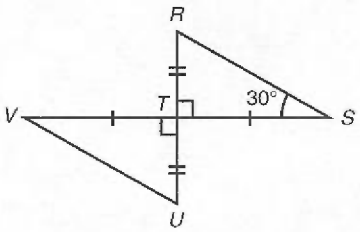


B. Answer each question. If you do not have enough information to solve a problem, write *Not enough information is given.*

Refer to the figure below to answer Questions 5 through 7.

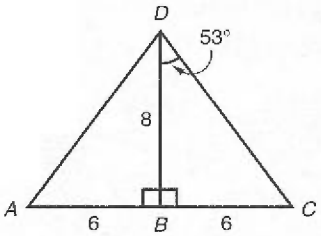


5. Is $\triangle TRS \cong \triangle TUV$? How do you know?

6. What is the measure of $\angle V$?

7. What is the measure of $\angle U$?

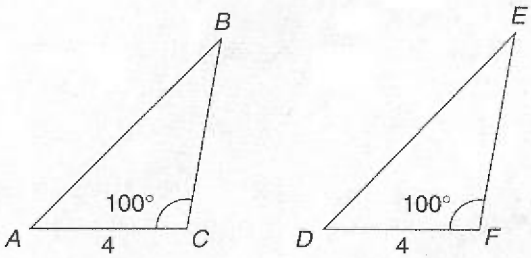
Refer to the figure below to answer Questions 8 and 9.



8. Is $\triangle BDA \cong \triangle BDC$? How do you know?

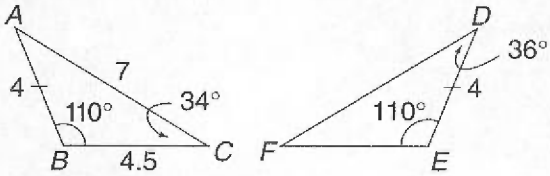
9. What is the measure of $\angle A$?

Refer to the figure below to answer Question 10.



10. Is $\triangle ABC \cong \triangle DEF$? How do you know?

Refer to the figure below to answer Questions 11 and 12.



11. What is the measure of $\angle A$?

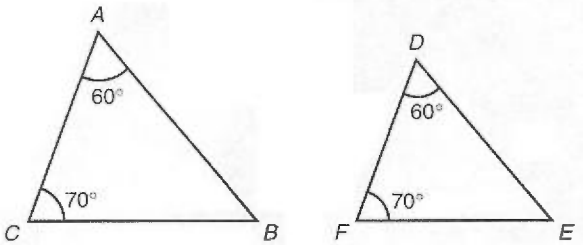
12. What is the length of $\overline{EF}$?

Answers start on page 441.

Similar Figures

Two figures are **similar** (~) if their corresponding angles have equal measure and the corresponding sides are in proportion. Similar figures have the same shape, but they are *not necessarily* the same size.

If the measures of two angles of one triangle are equal to two angle measures in another triangle, the measures of the third angles will also be equal (AAA) and the triangles are similar.

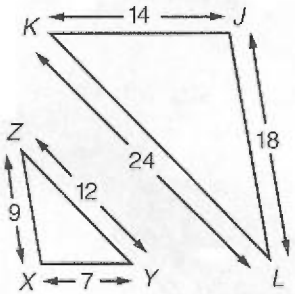


Example 1 Is $\triangle ABC$ similar to $\triangle DEF$?

In $\triangle ABC$, $m\angle A = 60°$. In $\triangle DEF$, $m\angle D = 60°$.
In $\triangle ABC$, $m\angle C = 70°$. In $\triangle DEF$, $m\angle F = 70°$.

Since the measures of two angles of $\triangle ABC$ are equal to two angle measures in $\triangle DEF$, the triangles are similar. $\boldsymbol{\triangle ABC \sim \triangle DEF}$

If the lengths of the sides of one triangle are proportional to the lengths of the sides of the other triangle, the triangles are similar.



Example 2 Is $\triangle XYZ$ similar to $\triangle JKL$?

In $\triangle XYZ$, $\overline{XY}$ measures 7. In $\triangle JKL$, $\overline{JK}$ measures 14.
In $\triangle XYZ$, $\overline{XZ}$ measures 9. In $\triangle JKL$, $\overline{JL}$ measures 18.
In $\triangle XYZ$, $\overline{YZ}$ measures 12. In $\triangle JKL$, $\overline{KL}$ measures 24.

Write ratios comparing a side in $\triangle XYZ$ to its corresponding side in $\triangle JKL$. Reduce to simplest terms.

$\frac{7}{14} = \frac{9}{18} = \frac{12}{24}$ Each ratio equals $\frac{1}{2}$; they are equal.

Since the ratios are equal, the lengths of the sides of $\triangle XYZ$ are proportional to the lengths of the sides of $\triangle JKL$. $\boldsymbol{\triangle XYZ \sim \triangle JKL}$

TIP

If the second triangle is turned differently than the first one, redraw one of the triangles to make it easier to see which sides correspond.

Similar triangles are often used to solve problems when it is not possible to find a distance by measuring.

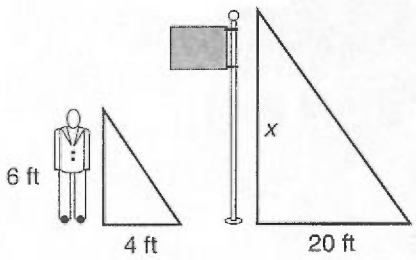
Example 3 At 4 p.m., a flagpole casts a 20-foot shadow. At the same time, a person 6 feet tall casts a 4-foot shadow. What is the height of the flagpole?

The sun strikes the person and flagpole at the same angle (since measurements are taken at the same place and same time). So, the objects and shadows form similar triangles.

Set up a proportion. $\frac{\text{person's height}}{\text{flagpole's height}} \quad \frac{6}{x} = \frac{4}{20} \quad \frac{\text{person's shadow}}{\text{flagpole's shadow}}$

Solve.

$$4x = 6(20)$$
$$x = 120 \div 4$$
$$x = 30$$

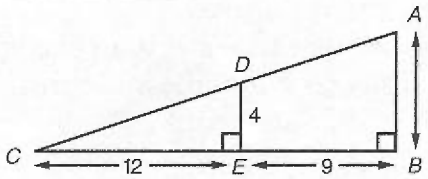


The height of the flagpole is **30 feet.**

GED SKILL FOCUS

Solve.

Refer to the figure below to answer Questions 1 and 2.

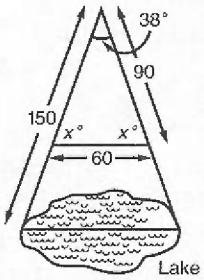


1. In the figure, $\triangle ABC \sim \triangle DEC$. Side $\overline{EC}$ is similar to which side of $\triangle ABC$?

2. What is the length of $\overline{AB}$?

Refer to the information and figure below to answer Questions 3 and 4.

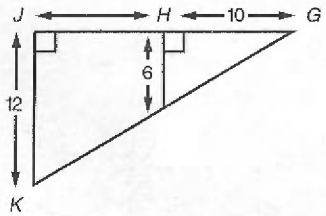
To determine the width of a lake, surveyors measure off two similar isosceles triangles as shown.



3. What is the value of x?

4. What is the distance across the longest part of the lake?

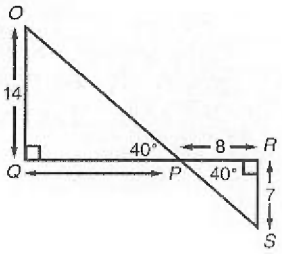
Refer to the figure below to answer Questions 5 and 6.



5. In the figure, $\triangle GHI$ and $\triangle GJK$ are similar. What is the length of $\overline{GJ}$?

6. The measure of $\angle GIH$ is equal to the measure of what other angle?

Refer to the figure below to answer Questions 7 and 8.



7. $\angle S$ has the same measure as what other angle?

8. What is the length of $\overline{PQ}$?

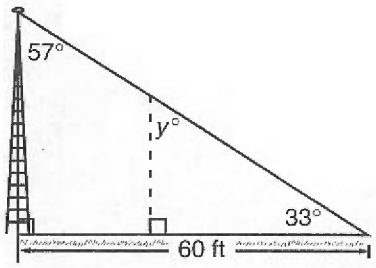
---

9. At 11 A.M., a 5-foot post casts a 3-foot shadow. At the same time, a tree casts a shadow that is 21 feet in length. What is the height of the tree?

10. At 6 P.M., a signpost casts a shadow that is 4 feet in length. At the same time, a street lamp casts a shadow that is 16 feet in length. If the signpost is 6 feet tall, what is the height of the street lamp?

Refer to the information and figure below to answer Questions 11 and 12.

A 42-foot tower has a diagonal brace. To reduce the effects of the wind, an engineer wants to add a vertical support 20 feet from the tower and shown by the dotted line in the drawing.



11. What is the value of $\angle y$?

12. What will be the height of the new support?

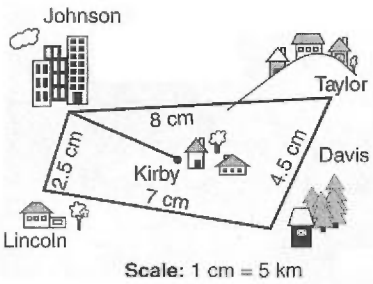
Answers start on page 442.

GED STRATEGY **Solving Word Problems**

Using Proportion in Geometry

Many measurement and geometry problems involve **indirect measurement.** Instead of making actual measurements, use proportions and knowledge of corresponding parts to find the answer. Finding a missing measurement when working with similar triangles is an example of indirect measurement. There are two other common measurement situations that can be solved using indirect measurement.

A **scale drawing** is a sketch of an object with all distances proportional to corresponding distances on the actual object. The **scale** gives the ratio of the sketch measurements to the actual measurements. A map is an example of a scale drawing.



Example 1 The distance on the map between Taylor and Davis is 4.5 cm. What is the actual distance between the two towns?

Step 1 **Read the map scale.** According to the scale 1 centimeter on the map equals 5 kilometers of actual distance.

Step 2 **Write a proportion.** $\frac{\text{map distance}}{\text{actual distance}} \quad \frac{1 \text{ cm}}{5 \text{ km}} = \frac{4.5 \text{ cm}}{x \text{ km}}$

Step 3 **Solve.** $x = 5(4.5) = 22.5$ km

The actual distance is **22.5 kilometers.**

TIP

With real-life situations, use experience to decide whether your answers make sense. Is it reasonable that a living room would measure 12 ft by 18 ft? Yes, it is.

A **floor plan** is another example of indirect measurement. A floor plan is a map showing the layout of the rooms in a building. A floor plan usually shows the placement of doors and windows. It may also use symbols to show the placement of furniture.

Example 2 In a floor plan of the Martin's house, the rectangular living room measures 3 inches by $4\frac{1}{2}$ inches. Every 2 inches on the floor plan represents 8 actual feet. How many square feet of carpet are needed to cover the living room floor?

Step 1 **Read the scale.** The scale is given in the problem.
2 inches = 8 feet

Step 2 **Write the proportions.** You need to write two proportions to find the actual length and width of the room.

$$\frac{2 \text{ inches}}{8 \text{ feet}} = \frac{4.5 \text{ inches}}{l \text{ feet}} \qquad \frac{2 \text{ inches}}{8 \text{ feet}} = \frac{3 \text{ inches}}{w \text{ feet}}$$

Step 3 **Solve.**

$$2l = 8(4.5) \qquad 2w = 8(3)$$
$$2l = 36 \qquad 2w = 24$$
$$l = 18 \text{ feet} \qquad w = 12 \text{ feet}$$

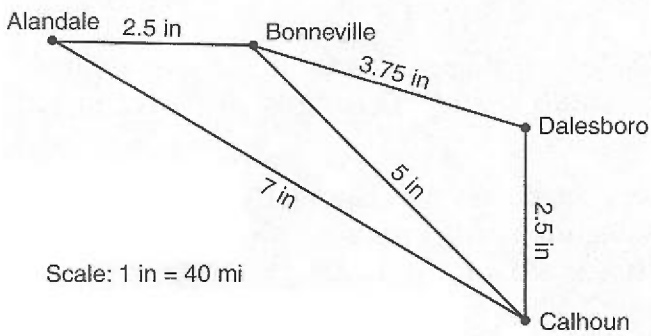
Step 4 **Find the area.** $A = l \times w \quad 12 \times 18 = 216$ square feet

The living room floor will require **216 square feet** of carpet.

GED PRACTICE

Directions: Choose the one best answer to each question.

Questions 1 and 2 refer to the following drawing.



1. The map distances between cities are shown on the map. What is the actual distance in miles between Bonneville and Dalesboro?

   (1) 120
   (2) 150
   (3) 160
   (4) 180
   (5) 200

2. Susan drove from Calhoun to Alandale, then from Alandale to Bonneville, and back to Calhoun. How many miles did she drive?

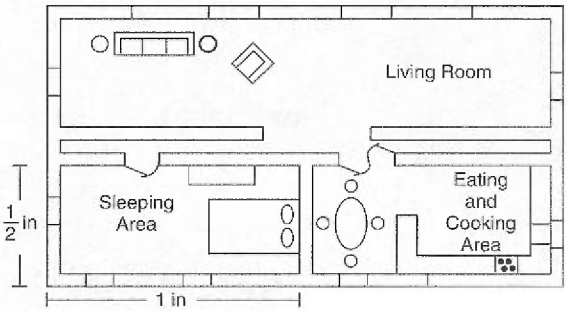
   (1) 460
   (2) 480
   (3) 540
   (4) 580
   (5) 620

---

3. Hillsboro is 50 miles from Merville. On a map these towns are 2.5 inches apart. What is the scale of this map?

   (1) 1 in = $\frac{1}{5}$ mi
   (2) 1 in = 2 mi
   (3) 1 in = 2.5 mi
   (4) 1 in = 20 mi
   (5) 1 in = 25 mi

4. In this floor plan, the scale is $\frac{3}{4}$ in = 15 ft. What are the dimensions of the actual sleeping area?



   (1) 10 feet by 20 feet
   (2) 12 feet by 24 feet
   (3) 15 feet by 30 feet
   (4) 20 feet by 40 feet
   (5) 25 feet by 50 feet

5. A floor plan is drawn to a scale of 1 in = 8 ft. On the plan, one wall is $1\frac{3}{4}$ inches long. Will a 14-foot wall unit fit on this wall?

   (1) No, the wall unit is about 2 feet too long.
   (2) No, the wall unit is about 1 foot too long.
   (3) Yes, the wall unit fits exactly.
   (4) Yes, the wall unit fits with about 1 foot of extra space remaining.
   (5) Yes, the wall unit fits with about 2 feet of extra space remaining.

6. A map scale is 1 in = 1.8 mi. How far is the actual distance in miles between two points that are 3.5 inches apart on the map?

   (1) 0.5
   (2) 1.9
   (3) 3.5
   (4) 6.3
   (5) 35.0

Answers start on page 443.

20. **(1) $\angle 5$** Three angles in the figure are equal to $\angle 4$: $\angle 7$ (vertical angle), $\angle 2$ (corresponding angle), and $\angle 5$ (alternate exterior angle). Only $\angle 5$ is an answer option.

21. **(1) $m\angle 8 = m\angle 1$** Angles 1 and 8 are alternate exterior angles (on the outside of the parallel lines and on opposite sides of the transversal). Alternate exterior angles have the same measure. Options (2) and (3) are incorrect because the figure gives no information about specific measures of the angles. Option (4) is incorrect because the measures of angles 4 and 8 total 180°, making these angles supplementary, not complementary (totaling 90°). Option (5) is incorrect because the measures of angles 3 and 4 total 180°, making these angles supplementary, not congruent which is the meaning of the symbol ≅. (Supplementary angles that are congruent would be two right angles, which the figure does not indicate.)

22. **(4) 10π** The circumference of a circle is equal to pi (π) × diameter. The radius of the circle is 5 cm; therefore, the diameter must be 2(5), or 10 cm. Substituting 10 for diameter, gives $C = \pi(10)$, or 10π. Remember, the order in which terms are multiplied does not affect the result.

23. **(3) $\angle FGC$ and $\angle DGE$ are vertical angles.** Option (1) is true, but it does not help you know the measure of $\angle FGC$. Option (2) is not true. You know the sum of the angles mentioned is not 180° because $\angle FGA$ is not a straight angle. Option (4) is not true because the angles are not positioned in relation to a transversal. Option (5) is true, but this fact has no bearing on the measure of $\angle FGC$. Only option (3) provides proof that $\angle FGC$ measures 75°. Since $\angle DGE$ measures 75° and vertical angles are equal, $m\angle FGC = 75°$.

24. **(1) 9** Find the height of the triangle that has an area = 90 and base = 20. Use the area formula.

Formula: $A = \frac{1}{2} \times \text{base} \times \text{height}$

Substitute values: $90 = \frac{1}{2}(20)h$

Solve for h: $90 = 10h$

$9 = h$

Lesson 25

GED Skill Focus (Page 299)

1. **$\triangle ABC$** (or **$\triangle ACB$** or **$\triangle CBA$**) Each side has the same length, 10. Note: The order of the letters in the name of the triangle does not matter.

2. **$\triangle ABD$** Two of its sides have the same length, 13. (The equilateral triangle, $\triangle ABC$, is a special case of an isosceles triangle.)

3. **$\triangle ACD$ and $\triangle BCD$** In each triangle, there is one angle that is obtuse, that is, greater than 90°.

4. **$\triangle ABE$, $\triangle ABC$, and $\triangle ABD$** All the angles are acute angles, that is, less than 90°.

5. **$\triangle ABE$, $\triangle ACD$, and $\triangle BCD$** No sides are equal.

6. **$\triangle ABE$**

7. **$\triangle BDC$ and $\triangle CDE$**

8. **$\triangle ACB$ and $\triangle BCE$**

9. **35°** The sum of the measures of all three angles of $\triangle ABE$ must be equal to 180°.
$$m\angle A + m\angle ABE + m\angle E = 180°$$
Since $m\angle A = 55°$ and $m\angle ABE = 90°$, then
$$m\angle E = 180° - 55° - 90° = 35°$$

10. **35°** In $\triangle ABC$ it is given that $m\angle ACB = 90°$ and $m\angle A = 55°$. The measures total 145°. Then, $m\angle ABC = 180° - 145° = 35°$

11. **55°** In $\triangle DCE$, $m\angle CDE = 90°$ and $m\angle E = 35°$ (see question 9), which totals 125°. Then, $m\angle DCE = 180° - 125° = 55°$

12. **55°** In $\triangle BCE$, $m\angle BCE = 90°$ and $m\angle E = 35°$ (see question 9), which totals 125°. Then, $m\angle CBD = 180° - 125° = 55°$. Also, since $m\angle ABE = 90°$ and $m\angle ABC = 35°$ (from question 10), then $m\angle CBD = 90° - 35° = 55°$

13. **35°** In $\triangle BCD$, $m\angle BDC = 90°$ and $m\angle CBD = 55°$ (see question 12), which totals 145°. Then, $m\angle BCD = 180° - 145° = 35°$. Also, since $m\angle BCE = 90°$ and $m\angle DCE = 55°$ (see question 11), $m\angle BCD = 90° - 55° = 35°$

14. All five triangles are right triangles. They also have the same angle measures: 35°, 55°, and 90°.

15. **4** A triangle with two sides of the same length is isosceles. $\triangle AEB$ has two sides of length 6.
$\triangle CED$ has two sides of length 3.
$\triangle ACD$ has two sides of length 5.
$\triangle CDB$ has two sides of length 5.

16. **18°** Since the triangle is a right triangle, one angle measures 90°. The measures of the other angles can be represented by x and $4x$. Write an equation and solve.
$$x + 4x + 90° = 180°$$
$$5x + 90° = 180°$$
$$5x = 90°$$
$$x = 18°$$

17. **41°, 57°, and 82°** Let x = the smallest angle measure. The other angle measures can be represented by $x + 16°$ and $2x$. Write an equation and solve.
$$x + x + 16° + 2x = 180°$$
$$16° + 4x = 180°$$
$$4x = 164°$$
$$x = 41°$$
Substitute to find the other measures.
$$x + 16° = 41° + 16° = 57°$$
$$2x = 2(41°) = 82°$$

18. **False** Each angle in an equilateral triangle measures 60°. Since $\angle B$ is a right angle, it must measure 90°.

19. **False** One angle in an obtuse triangle must be greater than 90°. In this triangle, the largest angle is $\angle B$, which measures 90°.

20. **True** Triangles ABC and BCD both have one right angle and an angle equal to 21°. In both triangles, the measure of the third angle must be $180° - 90° - 21° = 69°$.

21. **False** $\triangle ABD$ is smaller than the other triangles in the figure, but its angle measures are the same. $\angle BDA$ must measure 90° since it is a supplement to $\angle BDC$, which measures 90°.
Since $m\angle ABC = 90°$ and $m\angle C = 21°$, and the sum of the three angles in $\triangle ABC = 180°$, then $180° - 90° - 21° = 69° = m\angle A$.
If $m\angle BDA = 90°$ and $m\angle BAD = 69°$, then $m\angle DBA = 21°$

22. **False** An acute triangle has three angles with measures less than 90°. $\triangle ABC$ has a right angle, so it is a right triangle, not an acute triangle.

23. **True** Angles ABC and BDC are right angles as indicated by the right angle symbol (the small square in the angle) on the figure. Since $\angle BDC$ is a right angle and is supplementary to $\angle BDC$ (that is their measures total 180°) $\angle BDC$ must also be a right angle.

GED Skill Focus (Page 301)

1. **rhombus, square, parallelogram, or rectangle** Remember, a rhombus is a special parallelogram and a square is a special rectangle.

2. **square, rectangle, rhombus, or parallelogram** Remember, a square is a kind of rhombus and a rectangle is a kind of parallelogram.

3. **parallelogram, square, rectangle, or rhombus** A trapezoid has only one pair of parallel opposite sides.

4. **trapezoid**

5. **parallelogram, rhombus, or trapezoid**

6. **trapezoid**

7. **trapezoid**

8. **360°** The sum of the inside angle measures of any quadrilateral is 360°.

9. **55°** Angles K and L each measure 125° for a total of 250°. The remaining angles must measure $360° - 250° = 110°$. Let x stand for the measure of angle J. Since the remaining angles are equal, $2x = 110°$ and $x = 55°$; therefore, $m\angle J = 55°$.

10. **parallelogram**

11. **65°** In a parallelogram, the opposite angles are equal; therefore, the measure of $\angle A$ is 65°, the same as the measure of $\angle C$.

12. **115°** If angles A and C each measure 65°, their sum is $2(65°) = 130°$. The sum of the remaining angles must be $360° - 130° = 230°$. Since the remaining angles have an equal measure, each measure is half of 230°, or 115°, $m\angle D = 115°$.

13. **square, rectangle, rhombus, parallelogram** The figure is either a square or a rectangle, but you should remember that a square is a kind of rhombus and a rectangle is a kind of parallelogram.

14. **90°, 90°, 100°, and 80°** You know that the sum of the four angles is 360°. Two angles are right angles, each measuring 90°. Let x = the measure of the larger unknown angle and $x - 20°$ = the measure of the smaller unknown angle. Write an equation and solve.
$$\begin{aligned} x + (x - 20°) + 90° + 90° &= 360° \\ 2x - 20° &= 180° \\ 2x &= 200° \\ x &= 100° \\ x - 20° = 100° - 20° &= 80° \end{aligned}$$
The angles are 90°, 90°, 100°, and 80°.

15. **parallelogram** The figure cannot be a rectangle because there are no right angles. It cannot be a rhombus or square because all sides are not equal in length. It cannot be a trapezoid because there are two pairs of opposite parallel sides.

16. **They are parallel, and they are the same length.** In a parallelogram, opposite sides are always parallel. (Note that the definition of a parallelogram gives no information about the measures of the angles. However, a parallelogram can be looked at as parallel lines cut by a transversal, so given the measure of one angle, the measures of the other angles can be determined.)

17. **130°** Angle AEB of the triangle and $\angle DEB$ of the parallelogram are supplementary; that is, their sum is 180°.
Since $m\angle AEB = 50°$, $m\angle DEB = 180° - 50° = 130°$.
In a parallelogram, the opposite angles are equal in measure, so $m\angle C = 130°$.

18. **90°** Since lines $\overline{AD}$ and $\overline{BC}$ are parallel and the transversal, line $\overline{AB}$, is perpendicular to $\overline{AB}$ (because $m\angle A = 90°$), $\overline{AB}$ is also perpendicular to $\overline{BC}$. Thus, $m\angle ABE = 90°$.

GED Skill Focus (Page 303)

1. **Yes** The triangles are equilateral and congruent. Since $\angle D$ and $\angle E$ each measure 60°, $\angle F$ must also measure 60°, making $\triangle DEF$ equilateral. You know that the three sides of $\triangle ABC$ are equal and that the sides are equal to the measure of side. If a triangle has three sides with equal measures, it is equilateral. Therefore, both are equilateral and congruent.

2. **No** The triangles have equal angle measures, but this is *not* proof that the triangles are congruent. Since $\overline{MO}$ does not *equal* the measure of its corresponding side $\overline{PR}$, the triangles cannot be congruent. They are the same *shape* but not the same *size*.

3. **Yes** The lengths of the corresponding sides are equal; therefore, the triangles are congruent according to the SSS rule.

4. **Yes** The lengths of two sides and the measure of the angle between them are equal for both triangles; therefore, the triangles are congruent according to the SAS rule.

5. **Yes** $\overline{TV}$ and $\overline{TS}$ have the same measure, $\overline{TR}$ and $\overline{TU}$, and the angles between each pair of sides have the same measures (90°). Thus, the triangles are congruent according to Rule SAS.

6. **30°** $\angle V$ corresponds to $\angle S$

7. **60°** If $m\angle V = 30°$ and $m\angle T = 90°$, then $m\angle U = 180° - 30° - 90° = 60°$

8. **Yes** $\overline{BD}$ is a side in both triangles. $\overline{AB}$ and $\overline{BC}$ have the same measure. In each triangle, the sides mentioned have a right angle between them. Therefore, the triangles are congruent according to the SAS rule.

9. **37°** $m\angle BDC = 53°$, so $m\angle C = 180° - 90° - 53° = 37°$ $\angle A$ corresponds to $\angle C$ and the triangles are congruent, so $m\angle A = 37°$

10. **Not enough information is given.** The triangles may or may not be congruent. You need either the measures of $\angle A$ and $\angle D$ or the measures of sides $\overline{CB}$ and $\overline{FE}$ to determine whether the triangles are congruent.

11. **36°** The sum of the angles of a triangle equals 180°. $180° - 110° - 34° = 36°$

12. **4.5** The triangles are congruent since $m\angle A = 36°$. (See question 11.) Since the triangles are congruent, side $\overline{EF}$ must equal 4.5.

GED Skill Focus (Page 305)

1. $\overline{BC}$

2. **7** Write a proportion and solve. Note that the length of side $\overline{BC} = 12 + 9 = 21$

$$\frac{12}{21} = \frac{4}{x}$$
$$12x = 4(21)$$
$$12x = 84$$
$$x = 7$$

3. **71°** An isosceles triangle has two angles with the same angle measures (x). Write an equation.

$$38° + 2x = 180°$$
$$2x = 142°$$
$$x = 71°$$

4. **100** Write a proportion and solve.

$$\frac{150}{90} = \frac{x}{60}$$
$$90x = 150(60)$$
$$90x = 9000$$
$$x = 100$$

5. **20** Write a proportion and solve.

$$\frac{6}{12} = \frac{10}{x}$$
$$6x = 12(10)$$
$$6x = 120$$
$$x = 20$$

6. $\angle GKJ$ Side GI corresponds to $\overline{GK}$, and $\overline{IH}$ corresponds to $\overline{KJ}$. The angles between these sides are congruent.

7. $\angle O$

8. **16** Write a proportion and solve.

$$\frac{7}{14} = \frac{8}{x}$$
$$7x = 14(8)$$
$$7x = 112$$
$$x = 16$$

9. **35 feet** Write a proportion and solve.

$$\frac{\text{post's height}}{\text{tree's height}} = \frac{\text{post's shadow}}{\text{tree's shadow}}$$
$$\frac{5}{x} = \frac{3}{21}$$
$$21(5) = 3x$$
$$105 = 3x$$
$$35 = x$$

10. **24 feet** Write a proportion and solve.

$$\frac{\text{signpost's height}}{\text{street lamp's height}} = \frac{\text{signpost's shadow}}{\text{street lamp's shadow}}$$
$$\frac{6}{x} = \frac{4}{16}$$
$$96 = 4x$$
$$24 = x$$

11. **57°** The ground, tower, and brace form a triangle. The new support will create another similar triangle inside the larger one. The angle from the new support to the brace corresponds to the angle from the tower to the brace.

12. **28 feet** The support is 20 ft from the tower. Thus, the base of the smaller triangle is $60 - 20 = 40$. Write a proportion and solve.

$$\frac{42}{x} = \frac{60}{40}$$
$$42(40) = 60x$$
$$1680 = 60x$$
$$28 = x$$

GED Practice (Page 307)

NOTE: Throughout this section, decimal values have been used for fractions. However, you can solve the same proportions using fractions. When using a calculator to solve problems, use decimals for common fractions such as

$$\frac{1}{4} = 0.25 \quad \frac{1}{2} = 0.5 \quad \frac{3}{4} = 0.75$$

1. **(2) 150** Write a proportion and solve.

$$\frac{1 \text{ in}}{40 \text{ mi}} = \frac{3.75 \text{ in}}{x \text{ mi}}$$
$$1x = 40(3.75)$$
$$x = 150 \text{ mi}$$

2. **(4) 580** Add the distances on the map between the cities. 7 in + 2.5 in + 5 in = 14.5 in. Write a proportion and solve.

$$\frac{1 \text{ in}}{40 \text{ mi}} = \frac{14.5 \text{ in}}{x \text{ mi}}$$
$$1x = 40(14.5)$$
$$x = 580 \text{ mi}$$

3. **(4) 1 in = 20 mi** Write a proportion and solve.

$$\frac{2.5 \text{ in}}{50 \text{ mi}} = \frac{1 \text{ in}}{x \text{ mi}}$$
$$2.5x = 50(1)$$
$$x = 20 \text{ mi}$$

4. **(1) 10 feet by 20 feet** Write proportions to find both

$$\frac{0.75 \text{ in}}{15 \text{ ft}} = \frac{0.5 \text{ in}}{x \text{ ft}}$$
$$0.75x = 15(0.5)$$
$$0.75x = 7.5$$
$$x = 10 \text{ ft}$$

$$\frac{0.75 \text{ in}}{15 \text{ ft}} = \frac{1 \text{ in}}{x \text{ ft}}$$
$$0.75x = 15(1)$$
$$0.75x = 15$$
$$x = 20 \text{ ft}$$

5. **(3) Yes, the wall unit fits exactly.** Write the ratios in the form of inches to feet and solve the proportion.

$$\frac{1 \text{ in}}{8 \text{ ft}} = \frac{1.75 \text{ in}}{x \text{ ft}}$$
$$1x = 8(1.75)$$
$$x = 14 \text{ ft}$$

6. **(4) 6.3** Write the ratios in the form of inches to miles. Then solve the proportion.

$$\frac{1 \text{ in}}{1.8 \text{ mi}} = \frac{3.5 \text{ in}}{x \text{ mi}}$$
$$1x = 1.8(3.5)$$
$$x = 6.3 \text{ mi}$$

Lesson 26

GED Skill Focus (Page 309)

1. **132 square feet** Think of the figure as a rectangle and a triangle. Find the area of each

Rectangle: $A = lw$
$= 8(4)$
$= 32$ sq ft

Triangle: $A = \frac{1}{2}bh$
$= \frac{1}{2}(10)(20)$
$= 100$ sq ft

Add the two areas. $32 + 100 = 132$ sq ft

2. **304.5 square inches** Think of the figure as two triangles and a rectangle. Find the area of each element and add the areas.

Left triangle: $A = \frac{1}{2}bh = \frac{1}{2}(14)(18) = 126$ sq in

Rectangle: $A = lw = 14(9) = 126$ sq in

Right triangle: $A = \frac{1}{2}bh = \frac{1}{2}(14)(7.5) = 52.5$ sq in

Add. $126 + 126 + 52.5 = 304.5$ sq in

3. **168.5 square inches** Think of the figure as a circle and a rectangle since the two half-circles can be combined to form a circle. Find the area of each element and add the results.

Circle: The diameter = 10 in; so, the radius = 5 in
$A = \pi r^2 = 3.14(5^2) = 3.14(25) = 78.5$ sq in

Rectangle: $A = lw = 10(9) = 90$ sq in

Add. $78.5 + 90 = 168.5$ sq in

4. **164 square feet** Think of the figure as two equal parallelograms and a square.

Parallelograms: $A = bh = 16(4) = 64$ sq ft

Square: $A = s^2 = 6^2 = 36$ sq ft

Add. $64 + 64 + 36 = 164$ sq ft

5. **1242 square inches** Think of the figure as two rectangles. Find the area of each and add.

Top rectangle: $A = lw = 54(15) = 810$ sq in

Bottom rectangle: $A = lw = 24(18) = 432$ sq in

Add. $810 + 432 = 1242$ sq in

6. **208 square inches** Think of the figure as three rectangles. Find the area of each element and add the results. Note that you must subtract the widths of the left and right rectangles from the total width of the figure to find the unknown dimension of the middle rectangle.

$20 - 3 - 5 = 12$ in

Left rectangle: $A = lw = 14(3) = 42$ sq in

Middle rectangle: $A = lw = 12(8) = 96$ sq in

Right rectangle: $A = lw = 14(5) = 70$ sq in

Add. $42 + 96 + 70 = 208$ sq in

UNIT 4